Explorit for Global Optimization

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Abstract

Optimization is ubiquitous. We propose an optimization algorithm inspired by a domain-general search process in viable organisms, who find valuable energy sources by persisting and switching over potential locations efficiently. Our approach enables the more simple, general and efficient method to search for optimal points in complex problems. Comparison in synthetic and challenging optimization problems shows noticeable improvements.

1 Introduction

Optimization is a significant problem for species: How to search for energy sources optimally defines the evolutionary and the adaptive success. Basically, organisms search for "entities" to satisfy a wide range of energy demands, whether it be primary nutrition or high level self-transcendence. Often, optimal search must regulate foci effectively: concentrating efforts over familiar energy sources (exploitation) or switching to novel but potentially richer ones (exploration) [1].

Exploitation is focused, transient and phasic: it chooses actions with low variability in light of past experience to benefit from existing resource contingencies (persistance, maintenance, perseverance, refinement). Physiologically, brain regions in the striatum and ventromedial prefrontal cortex perform this task [2].

Exploration is diffuse, slow and tonic: it chooses actions with high variability when past experiences do not lead to adequate progress (curiosity, novelty, play, innovation) [3]. Brain regions linked with executive control in the frontopolar cortex and intraparietal sulcus are active during this task [2].

The "dilemma" and "trade-off" for the effective regulation of exploration and exploitation is well understood when human or rodent brains compete for attentional focus, where dopamine and norepinephrine "neuroregulate" such competition through the basal ganglia [3, 4]. But no mechanism for effective regulation, thus efficient search, has been proposed [5].

Here we introduce a simple and domain-general computational approach for optimization problems inspired by how viable organisms search for energy sources. Basically, instead of competition, we consider exploration and exploitation being in cooperation. Thus, we assert that organisms that explore and exploit intensively find better energy sources faster and more efficiently. Our approach aims at contributing towards the generalized search theory.

2 Explorit on Near Optimal Search

We consider the general problem: Optimize \( f(x) : S \rightarrow \mathbb{R} \) from some set \( S \subset \mathbb{R} \). Our goal is to find \( x^* \) in \( S \) such that \( f(x^*) \) is better or equal than \( f(x), \forall x \in S \).
An organism is designed to maintain stable energy incomes by searching entities \( E \) in the space \( S \) with high degrees of value, quality and novelty. To guide its search, the organism uses a generalized heuristic and adaptive memory elements. Fig. 1 provides the basic idea of the relationship of these elements and details are described hereafter.

The **Search Space** \( S \) is given by the problem in hand, and the organism models \( S \) as a collection of entities \( E \), with cardinality \( |S| = \prod_{i=1}^{D} \eta^E_i \) and boundaries \( B_S = \{B^L_S, B^U_S\} \), where:

- \( \eta^E_i \) is the number of entities \( E \) in the \( i \)-th dimension, and \( D \) is the dimensionality of \( S \).
- \( B_S \) denotes the lower \((B^L_S)\) and upper \((B^U_S)\) limits of space \( S \), respectively.

The **Entity** \( E = \{F_E, B_E\} \) denotes a generalized "concept" or "idea" of a solution \( x \in S \). Assuming that the Organism searches in \( \mathbb{R} \), the entity \( E \) must consider the following elements:

- \( F_E \) is the set of referential points of entity \( E \) in \( \mathbb{R} \).
- \( B_E = \{B^L_E, B^U_E\} \) defines the lower \((B^L_E)\) and upper \((B^U_E)\) boundaries of entity \( E \).
- \( \text{Fitness}_E = \frac{1}{|F_E|} \sum_{g \in F_E} f(F^g_E) \) defines the fitness performance of entity \( E \).
- \( \text{Novelty}^R_E = \text{obj}(\text{Fitness}_E - Q_R) \) represents how better\(^2\) the entity \( E \) is compared to the referential set \( R \subset S \). \( Q_R \) is the fitness quantile \( q_R \) of the set \( R \).
- If \( E \cap R = \emptyset \) then \( \text{Novelty}^R_E = 1 \), else \( \text{Novelty}^R_E = 0 \).

Moreover, for the sake of simplicity and without loss of generality, we assume that:

- \( |F_E| = 1 \), thus \( F_E \in \mathbb{R} \).
- \( \forall \text{ dimension } i \text{ of } S, \eta^E_i = \eta^E \) and \( \forall \text{ entity } E \in S, d_E = d = (B^L_E - B^U_E)/\eta^E \).
- \( B^L_E = F_E + w_1 d \) and \( B^U_E = F_E + w_2 d \), where \( w_1 = w_2 = 1/2 \).

The **Organism** represents a computational agent provided with an heuristic(Algorithm 1) and memory elements \( M, P_1 \) and \( P_2 \) in order to "explorit" the search space \( S \). Concretely speaking, explorit means to explore and exploit entities \( E \) in \( S \) such that:

- The **organism is alive** if the average energy income is equal or greater than a tolerance \( c_{tol} \) in the last \( o_A \cdot |S| \) time steps \( t \), where \( o_A \in [0, 1] \).
- The **organism can search in** \( S \) if the average energy income is equal or greater than a tolerance \( c_{sa} \) in the last \( o_S \cdot |S| \) time steps \( t \), where \( o_S \in [0, 1] \) and \( o_S < o_A \), and the elapsed time since the last energy income is equal or greater than a tolerance \( t_{tol} \).

\(^2\text{obj} = 1\), maximization.
The search space and boundaries
while compute the order of entity
Both
The set of potential solutions
In
compute the reference
Explorit performs exploitation(c = 0) and exploration(c = 1)
for each E in P_{c+1} do
for each dimension i in S do
v ← ⌊3r_1⌋ – 1 \quad \text{// v is a random integer in [-1,1], r_1 is a random value U[0,1]}
g ← r_2c \quad \text{// r_2 is a random value U[0,1]}
k_E ← ||1 + (B_{E}^L - B_{E}^Z)/d|| \quad \text{// k_E is the order of entity E in space S}
s_L ← z_0[1 + g(k_E - 2)] \quad \text{// if } k_E > 1 \text{ then } z_L = 1, \text{ else } z_L = 0
s_U ← z_0[1 + g(n_E - k_E - 1)] \quad \text{// if } n_E > k_E \text{ then } z_U = 1, \text{ else } z_U = 0
k_{E'} ← k_E + \frac{w}{E(z + 1)s_U + (v - 1)s_L} \quad \text{// compute the order of entity E' in S}
F_{E'} ← B_{E}^Z + (k_{E'} - \frac{1}{2})d \quad \text{// compute the reference } F_{E'} \text{ of entity E'}
if Novelty_{M} > 0 \text{ then} \quad \text{// evaluate Novelty and Quality of E'. Update } M, P_1, P_2
\quad t ← t + 1
\quad \{ Add E' to P_2 \} \leftrightarrow \{ Quality_{M} > 0 \}
\quad \{ Add E' to M \}
\quad \{ Delete every E'' ∈ P_2 \} \leftrightarrow \{ Quality_{M} < 0 \}
\quad \{ Add every E'' ∈ P_2 to P_1 \} \leftrightarrow \{ Quality_{M} > 0 \}
\quad Update energy income c_i^n ← max(Quality_{M}^E) - c_{i}^{n-1}
\quad S' ← P_1
\end{algorithm}

- In exploitation, the organism focuses in locations close to the set of entities E that the memory P_1 suggests valuable energy has been found, where P_1 = \{ E ∈ S : Quality_{P_2} > 0 \} represents the set of valuable entities.
- In exploration, the organism focuses in locations far from the set of entities E that the memory P_2 suggests potential energy has been found, where P_2 = \{ E ∈ S : Novelty_{M} > 0 \land Quality_{M} > 0 \} defines the set of potential entities.
- Both exploration and exploitation build and update the memory M, P_1 and P_2 incrementally and adaptively\(^3\).

3 Experiments

To validate explorit, we compared performance with the state-of-the-art literature in terms of global optimization. Our implementations used Matlab on an Intel(R) Core(TM) i7 CPU @2.8GHz 4GB RAM. Reference benchmarks include only the best methods for each problem studied, other benchmarks are well described in the respective references. Results indicate average and standard deviation over 20 independent trials. The parameters\(^4\) were set considering problem size for the instances below. The adaptive tuning would be a more effective case. Performance represents distance from the global optima, unless otherwise stated.

The studied problems include the following:
\(^3\)P_1 ⊂ P_2 ⊂ M
\(^4\)n_E = 21, c_{i,old} = 10^{-5}, t_{i,old} = 5, a_A = 0.4, a_S = 0.2, q_R = 0.5
Table 1: Results comparing the proposed method and recent benchmarks in four problem instances

<table>
<thead>
<tr>
<th>INSTANCE</th>
<th>$D$</th>
<th>$Evaluations$</th>
<th>$Performance$</th>
<th>$Evaluations$</th>
<th>$Performance$</th>
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</thead>
<tbody>
<tr>
<td>Synthetic</td>
<td>2</td>
<td>48±15</td>
<td>2.12E-6±1.41E-6</td>
<td>87±18</td>
<td>1.29E-4±1.71E-4</td>
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<tr>
<td>Multi-dimensional</td>
<td>1000</td>
<td>3E6</td>
<td>1.62E9±1.5E8</td>
<td>3E6</td>
<td>1.15E11±5.12E11</td>
</tr>
<tr>
<td>Vehicle Powertrain</td>
<td>9</td>
<td>386±71</td>
<td>(5.18, 0.58, 0.24, 0.27)</td>
<td>1020±192</td>
<td>(5.59, 2.14, 0.25, 0.24)</td>
</tr>
<tr>
<td>Image Segmentation</td>
<td>8</td>
<td>72±15</td>
<td>42.72±22.74</td>
<td>121±18</td>
<td>33.16±10.68</td>
</tr>
</tbody>
</table>

- **Synthetic**: Minimization over 30 functions generated by Gaussian Process Kernels[6].
- **Multi-dimensional**: Minimization over 20 unimodal and multimodal problems[7]. The maximum number of evaluations is set to $3.10^6$. The global optima is 0 for all problems.
- **Vehicle Powertrain**: Optimal configuration (design and control parameters) in parallel hybrid electric vehicles[8]. *Advisor* is the vehicle simulation tool. Performance represents fuel consumption (l/100km), emissions of CO (g/km), HC (g/km) and NOx (g/km) in the UDDS driving cycle.

Table 1 shows the simulation comparisons with the state-of-the-art methods. *Explorit* achieves improved performance with equal or better number of evaluations. The main reason is that *explorit* avoids overfocusing in promising but local-optima areas. Instead, it searches intensively considering value, quality and novelty aspects, thus the search regions are the result of not only fitness improvements, but also information gains.

### 4 Conclusions and Future Work

How can one search optimally? We have proposed *explorit* as a generalized process of joint exploration and exploitation in search. A unique point of this paper is that search emerges from the interplay of processes looking at quite different things, i.e., freewill and direction, while sharing attentional focus through memory. The proposed scheme offers a simple, general and efficient method to tackle optimization problems. Future work will aim at developing executive control functions as a search process, where energy management and brain development are central issues.

### References


