Explorit for Global Optimization

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Abstract

Optimization is ubiquitous. We propose an optimization algorithm inspired by a domain-general search process in viable organisms, who find valuable energy sources by persisting and switching over potential locations efficiently. Our approach enables the more simple, general and efficient method to search for optimal points in complex problems. Comparison in synthetic and challenging optimization problems shows noticeable improvements.

1 Introduction

Optimization is a significant problem for species: How to search for energy sources optimally defines the evolutionary and the adaptive success. Basically, organisms search for "*entities*" to satisfy a wide range of energy demands, whether it be primary nutrition or high level self-transcendence. Often, optimal search must regulate foci effectively: concentrating efforts over familiar energy sources(*exploitation*) or switching to novel but potentially richer ones(*exploration*)[1].

Exploitation is focused, transient and phasic: it chooses actions with low variability in light of past experience to benefit from existing resource contingencies(persistance, maintainance, perseverance, refinement). Physiologically, brain regions in the striatum and ventromedial prefrontal cortex perform this task[2].

Exploration is diffuse, slow and tonic: it chooses actions with high variability when past experiences do not lead to adequate progress(curiosity, novelty, play, innovation)[3]. Brain regions linked with executive control in the frontopolar cortex and intraparietal sulcus are active during this task[2].

The "dilemma" and "trade-off" for the effective regulation of *exploration* and *exploitation* is well understood when human or rodent brains *compete* for attentional focus, where dopamine and norepinephrine "neuroregulate" such *competition* through the basal ganglia[1][3, 4]. But no mechanism for effective regulation, thus efficient search, has been proposed[5].

Here we introduce a simple and domain-general computational approach for optimization problems inspired by how viable organisms search for energy sources. Basically, instead of *competition*, we consider *exploration* and *exploitation* being in *cooperation*. Thus, we assert that organisms that explore and exploit intensively find better energy sources faster and more efficiently. Our approach aims at contributing towards the generalized search theory.

2 *Explorit* on Near Optimal Search

We consider the general problem: *Optimize*$_x$ given a fitness function $f(x) : S \mapsto I\!R$ from some set $S \subset I\!R$. Our goal is to find x^* in S such that $f(x^*)$ is better or equal than $f(x), \forall x \in S$.

[1]Softmax temperature in Reinforcement Learning.

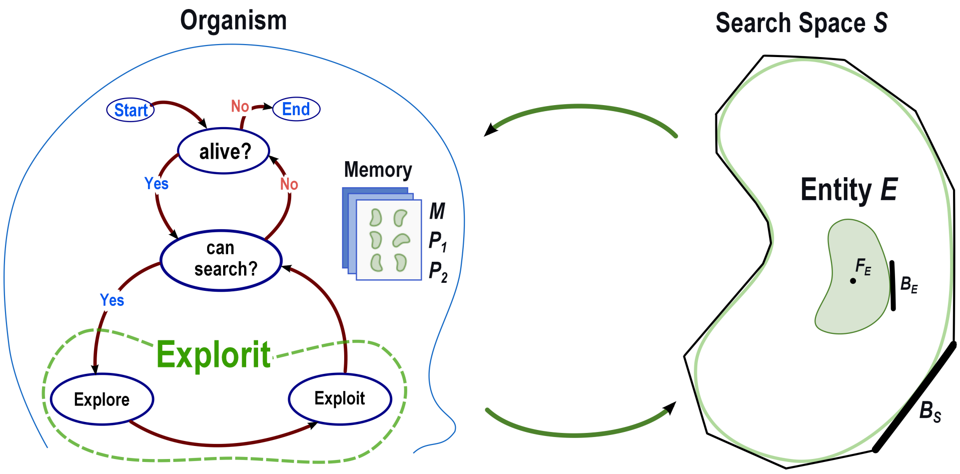

Figure 1: Basic structure of search with Explorit

An *organism* is designed to maintain stable energy incomes by searching *entities* E in the *space* S with high degrees of value, quality and novelty. To guide its search, the *organism* uses a generalized heuristic and adaptive memory elements. Fig. 1 provides the basic idea of the relationship of these elements and details are described hereafter.

The **Search Space** S is given by the problem in hand, and the *organism* models S as a collection of *entities* E, with cardinality $|S| = \prod_{i=1}^{D} \eta_i^E$ and boundaries $B_S = \{B_S^L, B_S^U\}$, where:

- η_i^E is the number of *entities* E in the i-th dimension, and D is the dimensionality of S.
- B_S denotes the *lower* (B_S^L) and *upper* (B_S^U) limits of *space* S, respectively.

The **Entity** $E = \{F_E, B_E\}$ denotes a generalized "concept" or "idea" of a solution $x \in S$. Assuming that the *Organism* searches in $\mathbb{R}$, the entity E must consider the following elements:

- F_E is the set of referential points of *entity* E in $\mathbb{R}$.
- $B_E = \{B_E^L, B_E^U\}$ defines the *lower* (B_E^L) and *upper* (B_E^U) boundaries of *entity* E.
- $Fitness_E = \frac{1}{|F_E|} \sum_{g \in F_E} f(F_E^g)$ defines the fitness performance of *entity* E.
- $Quality_E^R = obj(Fitness_E - Q_R)$ represents how better[2] the *entity* E is compared to the referential set $R \subset S$. Q_R is the fitness quantile q_R of the set R.
- If $E \cap R = \emptyset$ then $Novelty_E^R = 1$, else $Novelty_E^R = 0$.

Moreover, for the sake of simplicity and without loss of generality, we assume that:

- $|F_E| = 1$, thus $F_E \in \mathbb{R}$.
- $\forall$dimension i of S, $\eta_i^E = \eta^E$ and $\forall$*entity* $E \in S$, $d_E = d = (B_S^L - B_S^U)/\eta^E$.
- $B_E^L = F_E - w_1 d$ and $B_E^U = F_E + w_2 d$, where $w_1 = w_2 = 1/2$.

The **Organism** represents a computational agent provided with an heuristic(Algorithm 1) and memory elements M, P_1 and P_2 in order to "explorit" the *search space* S. Concretely speaking, *explorit* means to **explor**e and exploi**t** *entities* E in S such that:

- The *organism is alive* if the average energy income is equal or greater than a tolerance e_{tol} in the last $o_A.|S|$ time steps t, where $o_A \in [0, 1]$.
- The *organism can search in* S if the average energy income is equal or greater than a tolerance e_{tol} in the last $o_S.|S|$ time steps t, where $o_S \in [0, 1]$ and $o_S < o_A$, and the elapsed time since the last energy income is equal or greater than a tolerance t_{tol}.

[2] $obj = 1$, maximization.

Algorithm 1: Search Algorithm with *Explorit*

Input : *Search Space* S, *Boundaries* B_S^L and B_S^U // *The search space and boundaries*
Output: $S^{'}$ // *The set of potential solutions*

$S^{'} \leftarrow S, t \leftarrow 0, e_0^{in} \leftarrow 0$
while *Organism is alive* **do**
  Transfer focus to $S^{'}$ // $S \leftarrow S^{'}$, *clean* P_1, P_2 *and* M. *Initialize a random* $E \in S$
  while *Organism can search in* S **do**
    // $S^{'} \leftarrow Explorit(S)$
    for $c = 0$ *to* 1 **do** // *Explorit performs exploitation(*$c = 0$*) and exploration(*$c = 1$*)*
      for *each* E *in* P_{c+1} **do**
        for *each dimension* i *in* S **do**
          $v \leftarrow \lfloor 3r_1 \rfloor - 1$ // v *is a random integer in [-1,1],* r_1 *is a random value* U*[0,1]*
          $g \leftarrow r_2 c$ // r_2 *is a random value* U*[0,1]*
          $k_E \leftarrow \|1 + (B_E^L - B_S^L)/d\|$ // k_E *is the order of entity* E *in space* S
          $s_L \leftarrow z_L[1 + g(k_E - 2)]$ // *if* $k_E > 1$ *then* z_L *= 1, else* z_L *= 0*
          $s_U \leftarrow z_U[1 + g(\eta^E - k_E - 1)]$ // *if* $\eta^E > k_E$ *then* z_U *= 1 else* z_U *= 0*
          $k_{E'} \leftarrow k_E + \frac{|v|}{2}[(v+1)s_U + (v-1)s_L]$ // *compute the order of entity* $E^{'}$ *in* S
          $F_{E'} \leftarrow B_S^L + (k_{E'} - \frac{1}{2})d$ // *compute the reference* $F_{E'}$ *of entity* $E^{'}$
        if $Novelty_{E'}^M > 0$ **then** // *evaluate Novelty and Quality of* $E^{'}$. *Update* M, P_1, P_2
          $t \leftarrow t + 1$
          $\{$ *Add* $E^{'}$ *to* $P_2\} \leftrightarrow \{Quality_{E'}^M > 0\}$
          $\{$*Add* $E^{'}$ *to* $M\}$
          $\{$ *Delete every* $E^{''} \in P_2\} \leftrightarrow \{Quality_{E''}^M < 0\}$
          $\{$ *Add every* $E^{''} \in P_2$ *to* $P_1\} \leftrightarrow \{Quality_{E''}^{P_2} > 0\}$
          Update energy income $e_t^{in} \leftarrow max_E(Quality_E^M) - e_{t-1}^{in}$
    $S^{'} \leftarrow P_1$

- In *exploitation*, the *organism* focuses in locations close to the set of entities E that the memory P_1 suggests valuable energy has been found, where $P_1 = \{E \in S : Quality_E^{P_2} > 0\}$ represents the set of *valuable entities*.
- In *exploration*, the organism focuses in locations far from the set of entities E that the memory P_2 suggests potential energy has been found, where $P_2 = \{E \in S : Novelty_E^M > 0 \wedge Quality_E^M > 0\}$ defines the set of *potential entities*.
- Both *exploration* and *exploitation* build and update the memory M, P_1 and P_2 incrementally and adaptively[3].

3 Experiments

To validate *explorit*, we compared performance with the state-of-the-art literature in terms of *global* optimization. Our implementations used Matlab on an Intel(R) Core(TM) i7 CPU @2.8GHz 4GB RAM. Reference benchmarks include only the best methods for each problem studied, other benchmarks are well described in the respective references. Results indicate average and standard deviation over 20 independent trials. The parameters[4] were set considering problem size for the instances below. The adaptive tuning would be a more effective case. Performance represents distance from the global optima, unless otherwise stated.

The studied problems include the following:

[3] $P_1 \subset P_2 \subset M$
[4] $\eta^E = 21, e_{tol} = 10^{-5}, t_{tol} = 5, o_A = 0.4, o_S = 0.2, q_R = 0.5$

Table 1: Results comparing the proposed method and recent benchmarks in four problem instances

INSTANCE		EXPLORIT		BENCHMARK	
Name	D	**Evaluations**	**Performance**	**Evaluations**	**Performance**
Synthetic	2	48±15	2.12E-6±1.41E-6	87±18	1.29E-4±1.71E-4
Multi-dimensional	1000	3E6	1.62E9±1.5E8	3E6	1.15E11±5.12E11
Vehicle Powertrain	9	386±71	(5.18, 0.58, 0.24, 0.27)	1020±192	(5.59, 2.14, 0.25, 0.24)
Image Segmentation	8	72±15	42.72±22.74	121±18	33.16±10.68

- ***Synthetic***. Minimization over 30 functions generated by Gaussian Process Kernels[6].
- ***Multi-dimensional***. Minimization over 20 unimodal and multimodal problems[7]. The maximum number of evaluations is set to 3.10^6. The global optima is 0 for all problems.
- ***Vehicle Powertrain***. Optimal configuration(design and control parameters) in parallel hybrid electric vehicles[8]. *Advisor* is the vehicle simulation tool. Performance represents fuel consumption(l/100km), emissions of CO (g/km), HC(g/km) and NOx(g/km) in the UDDS driving cycle.
- ***Image Segmentation***. Optimal parameter set for a cosegmentation algorithm[9] in MSRC and VOC2012 datasets. Performance represents the average accuracy over 21 classes(intersection/union metric).

Table 1 shows the simulation comparisons with the state-of-the-art methods. *Explorit* achieves improved performance with equal or better number of evaluations. The main reason is that *explorit* avoids overfocusing in promising but local-optima areas. Instead, it searches intensively considering value, quality and novelty aspects, thus the search regions are the result of not only fitness improvements, but also information gains.

4 Conclusions and Future Work

How can one search optimally? We have proposed *explorit* as a generalized process of joint *exploration* and *exploitation* in search. A unique point of this paper is that search emerges from the interplay of processes looking at quite different things, i.e., freewill and direction, while sharing attentional focus through memory. The proposed scheme offers a simple, general and efficient method to tackle optimization problems. Future work will aim at developing executive control functions as a search process, where energy management and brain development are central issues.

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